MOAA 2018 Individual Round

1. Find $20 \cdot 18 + 20 + 18 + 1$.

2. Suzie's Ice Cream has 10 flavors of ice cream, 5 types of cones, and 5 toppings to choose from. An ice cream cone consists of one flavor, one cone, and one topping. How many ways are there for Sebastian to order an ice cream cone from Suzie's?

3. Let $a = 7$ and $b = 77$. Find $\frac{(2ab)^2}{(a+b)^2-(a-b)^2}$.

4. Sebastian invests 100,000 dollars. On the first day, the value of his investment falls by 20 percent. On the second day, it increases by 25 percent. On the third day, it falls by 25 percent. On the fourth day, it increases by 60 percent. How many dollars is his investment worth by the end of the fourth day?

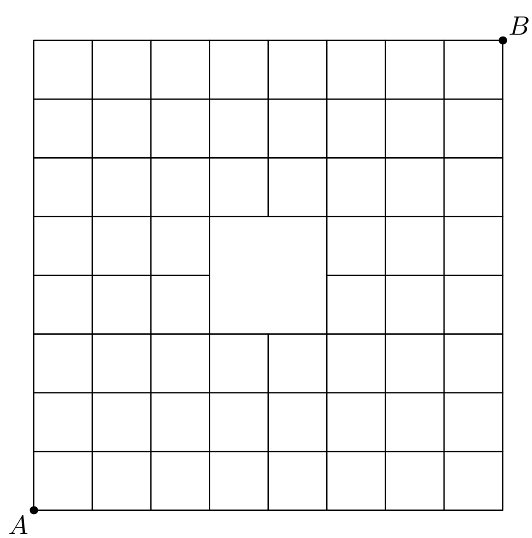
5. Square $ABCD$ has side length 5. Points K, L, M, N are on segments AB, BC, CD, DA respectively, such that $MC = CL = 2$ and $NA = AK = 1$. The area of trapezoid $KLMN$ can be expressed as $\frac{m}{n}$ for relatively prime positive integers m and n. Find $m + n$.

6. Suppose that p and q are prime numbers. If $p + q = 30$, find the sum of all possible values of pq.

7. Tori receives a $15 - 20 - 25$ right triangle. She cuts the triangle into two pieces along the altitude to the side of length 25. What is the difference between the areas of the two pieces?

8. The *factorial* of a positive integer n, denoted $n!$, is the product of all the positive integers less than or equal to n. For example, $1! = 1$ and $5! = 120$. Let $m!$ and $n!$ be the smallest and largest factorial ending in exactly 3 zeroes, respectively. Find $m + n$.

9. Sam is late to class, which is located at point B. He begins his walk at point A and is only allowed to walk on the grid lines. He wants to get to his destination quickly; how many paths are there that minimize his walking distance?



10. Mr. Iyer owns a set of 6 antique marbles, where 1 is red, 2 are yellow, and 3 are blue. Unfortunately, he has randomly lost two of the marbles. His granddaughter starts drawing the remaining 4 out of a bag without replacement. She draws a yellow marble, then the red marble. Suppose that the probability that the next marble she draws is blue is equal to $\frac{m}{n}$, where m and n are relatively prime positive integers. What is $m+n$?

11. If a is a positive integer, what is the largest integer that will always be a factor of $(a^3+1)(a^3+2)(a^3+3)$?

12. What is the largest prime number that is a factor of 160,401?

13. For how many integers m does the equation $x^2+mx+2018=0$ have no real solutions in x?

14. What is the largest palindrome that can be expressed as the product of two two-digit numbers? A palindrome is a positive integer that has the same value when its digits are reversed. An example of a palindrome is 7887887.

15. In circle ω inscribe quadrilateral $ADBC$ such that $AB \perp CD$. Let E be the intersection of diagonals AB and CD, and suppose that $EC=3, ED=4$, and $EB=2$. If the radius of ω is r, then $r^2=\frac{m}{n}$ for relatively prime positive integers m and n. Determine $m+n$.

16. Suppose that a, b, c are nonzero real numbers such that $2a^2+5b^2+45c^2=4ab+6bc+12ca$. Find the value of
$$\frac{9(a+b+c)^3}{5abc}.$$

17. Call a positive integer n *spicy* if there exist n distinct integers $k_1, k_2, \ldots, k_n$ such that the following two conditions hold:
    - $|k_1|+|k_2|+\cdots+|k_n|=n^2$,
    - $k_1+k_2+\cdots+k_n=0$.

    Determine the number of spicy integers less than 10^6.

18. Consider the system of equations
$$|x^2-y^2-4x+4y|=4$$
$$|x^2+y^2-4x-4y|=4.$$
    Find the sum of all x and y that satisfy the system.

19. Determine the number of 8 letter sequences, consisting only of the letters W,Q,N, in which none of the sequences WW, QQQ, or NNNN appear. For example, WQQNNNQQ is a valid sequence, while WWWQNQNQ is not.

20. Triangle $\triangle ABC$ has $AB=7$, $CA=8$, and $BC=9$. Let the reflections of A, B, C over the orthocenter H be A', B', C'. The area of the intersection of triangles ABC and $A'B'C'$ can be expressed in the form $\frac{a\sqrt{b}}{c}$, where b is squarefree and a and c are relatively prime. determine $a+b+c$. (The orthocenter of a triangle is the intersection of its three altitudes.)