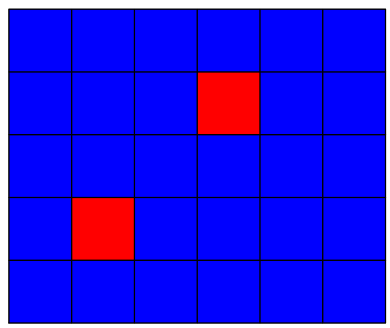
# MOAA 2019: Accuracy Round

1. Farmer John wants to bring some cows to a pasture with grass that grows at a constant rate. Initially, the pasture has some nonzero amount of grass and it will stop growing if there is no grass left. The pasture sustains 100 cows for ten days. The pasture can also sustain 100 cows for five days, and then 120 cows for three more days. If cows eat at a constant rate, find the maximum number of cows Farmer John can bring to the pasture so that they can be sustained indefinitely.

2. Sam is learning basic arithmetic. He may place either the operation $+$ or $-$ in each of the blank spots between the numbers below:
$$5 \_ 8 \_ 9 \_ 7 \_ 2 \_ 3.$$
In how many ways can he place the operations so the result is divisible by 3?

3. Will loves the color blue, but he despises the color red. In the $5 \times 6$ rectangular grid below, how many rectangles are there containing at most one red square and with sides contained in the gridlines?



4. Let $r_1, r_2, r_3$ be the three roots of a cubic polynomial $P(x)$. Suppose that
$$\frac{P(2)+P(-2)}{P(0)} = 200.$$
If $\frac{1}{r_1 r_2} + \frac{1}{r_2 r_3} + \frac{1}{r_3 r_1} = \frac{m}{n}$ for relatively prime positive integers $m$ and $n$, compute $m+n$.

5. Consider a rectangle $ABCD$ with $AB = 3$ and $BC = 1$. Let $O$ be the intersection of diagonals $AC$ and $BD$. Suppose that the circumcircle of $\triangle ADO$ intersects line $AB$ again at $E \neq A$. Then, the length $BE$ can be written as $\frac{m}{n}$ for relatively prime positive integers $m$ and $n$. Find $m+n$.

6. Let $ABCD$ be a square with side length 100 and $M$ be the midpoint of side $AB$. The circle with center $M$ and radius 50 intersects the circle with center $D$ and radius 100 at point $E$. $CE$ intersects $AB$ at $F$. If $AF = \frac{m}{n}$ for relatively prime positive integers $m$ and $n$, find $m+n$.

7. How many pairs of real numbers $(x, y)$, with $0 < x, y < 1$ satisfy the property that both $3x + 5y$ and $5x + 2y$ are integers?

8. Sebastian is coloring a circular spinner with 4 congruent sections. He randomly chooses one of four colors for each of the sections. If two or more adjacent sections have the same color, he fuses them and considers them as one section. (Sections meeting at only one point are not adjacent.) Suppose that the expected number of sections in the final colored spinner is equal to $\frac{m}{n}$ for relatively prime positive integers $m$ and $n$. Compute $m+n$.

9. Let $ABC$ be a triangle and $D$ be a point on the extension of segment $BC$ past $C$. Let the line through $A$ perpendicular to $BC$ be $\ell$. The line through $B$ perpendicular to $AD$ and the line through $C$ perpendicular to $AD$ intersect $\ell$ at $H_1$ and $H_2$, respectively. If $AB = 13$, $BC = 14$, $CA = 15$, and $H_1 H_2 = 1001$, find $CD$.

10. Find the sum of all positive integers $k$ such that
$$\frac{2}{1} - \frac{3}{2 \times 1} + \frac{4}{3 \times 2 \times 1} + \cdots + (-1)^{k+1} \frac{k+1}{k \times (k-1) \times \cdots \times 2 \times 1} \geq 1 + \frac{1}{700^3}.$$

# MOAA 2019: Accuracy Round

60 minutes

Name: ____________________________________________

Team Name: ________________________________________

Write your answers in the spaces provided below. All answers are integers between 0 and 1,000,000, inclusive.

| | | | |
|---|---|---|---|
| 1 | | 2 | |
| 3 | | 4 | |
| 5 | | 6 | |
| 7 | | 8 | |
| 9 | | 10 | |