MOAA 2019: Speed Round

1. What is $20 \times 19 + 20 \div (2 - 7)$?

2. Will has three spinners. The first has three equally sized sections numbered $1, 2, 3$; the second has four equally sized sections numbered $1, 2, 3, 4$; and the third has five equally sized sections numbered $1, 2, 3, 4, 5$. When Will spins all three spinners, the probability that the same number appears on all three spinners is p. Compute $\frac{1}{p}$.

3. Three girls and five boys are seated randomly in a row of eight desks. Let p be the probability that the students at the ends of the row are both boys. If p can be expressed in the form $\frac{m}{n}$ for relatively prime positive integers m and n, compute $m + n$.

4. Jaron either hits a home run or strikes out every time he bats. Last week, his batting average was .300. (Jaron's *batting average* is the number of home runs he has hit divided by the number of times he has batted.) After hitting 10 home runs and striking out zero times in the last week, Jaron has now raised his batting average to .310. How many home runs has Jaron now hit?

5. Suppose that the sum
$$\frac{1}{1 \cdot 4} + \frac{1}{4 \cdot 7} + \cdots + \frac{1}{97 \cdot 100}$$
is expressible as $\frac{m}{n}$ for relatively prime positive integers m and n. Compute $m + n$.

6. Let $ABCD$ be a unit square with center O, and $\triangle OEF$ be an equilateral triangle with center A. Suppose that M is the area of the region inside the square but outside the triangle and N is the area of the region inside the triangle but outside the square, and let $x = |M - N|$ be the positive difference between M and N. If
$$x = \frac{1}{8}\left(p - \sqrt{q}\right)$$
for positive integers p and q, find $p + q$.

7. Find the number of seven-digit numbers such that the sum of any two consecutive digits is divisible by 3. For example, the number 1212121 satisfies this property.

8. There is a unique positive integer x such that x^x has 703 positive factors. What is x?

9. Let x be the number of digits in 2^{2019} and let y be the number of digits in 5^{2019}. Compute $x + y$.

10. Let ABC be an isosceles triangle with $AB = AC = 13$ and $BC = 10$. Consider the set of all points D in three-dimensional space such that BCD is an equilateral triangle. This set of points forms a circle ω. Let E and F be points on ω such that AE and AF are tangent to ω. If EF^2 can be expressed in the form $\frac{m}{n}$, where m and n are relatively prime positive integers, determine $m + n$.

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20 minutes

Name: _____

Team Name: _____

Write your answers in the spaces provided below. All answers are integers between 0 and 1,000,000, inclusive.

1		2	
3		4	
5		6	
7		8	
9		10	