MOAA 2019: Team Round

1. **[45]** Jeffrey stands on a straight horizontal bridge that measures 20000 meters across. He wishes to place a pole vertically at the center of the bridge so that the sum of the distances from the top of the pole to the two ends of the bridge is 20001 meters. To the nearest meter, how long of a pole does Jeffrey need?

2. **[50]** The lengths of the two legs of a right triangle are the two distinct roots of the quadratic
$$x^2 - 36x + 70.$$
What is the length of the triangle's hypotenuse?

3. **[55]** For how many ordered pairs of positive integers (a, b) such that $a \le 50$ is it true that $x^2 - ax + b$ has integer roots?

4. **[60]** Brandon wants to split his orchestra of 20 violins, 15 violas, 10 cellos, and 5 basses into three distinguishable groups, where all of the players of each instrument are indistinguishable. He wants each group to have at least one of each instrument and for each group to have more violins than violas, more violas than cellos, and more cellos than basses. How many ways are there for Brandon to split his orchestra following these conditions?

5. **[65]** Let ABC be a triangle with $AB = AC = 10$ and $BC = 12$. Define ℓ_A as the line through A perpendicular to $\overline{AB}$. Similarly, ℓ_B is the line through B perpendicular to $\overline{BC}$ and ℓ_C is the line through C perpendicular to $\overline{CA}$. These three lines ℓ_A, ℓ_B, ℓ_C form a triangle with perimeter $\frac{m}{n}$ for relatively prime positive integers m and n. Find $m + n$.

6. **[70]** Let $f(x, y) = \lfloor \frac{5x}{2y} \rfloor + \lceil \frac{5y}{2x} \rceil$. Suppose x, y are chosen independently uniformly at random from the interval $(0, 1]$. Let p be the probability that $f(x, y) < 6$. If p can be expressed in the form $\frac{m}{n}$ for relatively prime positive integers m and n, compute $m + n$.

   (Note: $\lfloor x \rfloor$ is defined as the greatest integer less than or equal to x and $\lceil x \rceil$ is defined as the least integer greater than or equal to x.)

7. **[75]** Suppose ABC is a triangle inscribed in circle ω. Let A' be the point on ω so that AA' is a diameter, and let G be the centroid of ABC. Given that $AB = 13$, $BC = 14$, and $CA = 15$, let x be the area of triangle AGA'. If x can be expressed in the form $\frac{m}{n}$, where m and n are relatively prime positive integers, compute $100n + m$.

   (Note: The centroid of a triangle is the intersection of its three medians AM, BN, CP, where M, N, P are the midpoints of BC, CA, AB, respectively.)

8. **[80]** Suppose that
$$\frac{(\sqrt{2})^5 + 1}{\sqrt{2} + 1} \times \frac{2^5 + 1}{2 + 1} \times \frac{4^5 + 1}{4 + 1} \times \frac{16^5 + 1}{16 + 1} = \frac{m}{7 + 3\sqrt{2}}$$
for some integer m. How many 0's are in the binary representation of m? (For example, the number $20 = 10100_2$ has three 0's in its binary representation.)

9. **[85]** Jonathan finds all ordered triples (a, b, c) of positive integers such that $abc = 720$. For each ordered triple, he writes their sum $a + b + c$ on the board. (Numbers may appear more than once.) What is the sum of all the numbers written on the board?

10. **[90]** Let $\mathcal{S}$ be the set of all four digit palindromes (a *palindrome* is a number that reads the same forwards and backwards). The average value of $|m - n|$ over all ordered pairs (m, n), where m and n are (not necessarily distinct) elements of $\mathcal{S}$, is equal to $\frac{p}{q}$, for relatively prime positive integers p and q. Find $p + q$.

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60 minutes

Team Name: ______________________________

Problems in the Team Round are weighted based on the problem number. The value of each problem is shown in brackets next to each problem.

Write your answers in the spaces provided below. All answers are integers between 0 and 1,000,000, inclusive.

1		2	
3		4	
5		6	
7		8	
9		10	