MOAA 2021: Accuracy Round

October 16th, 2021

Rules

- No mathematical texts, notes, or online resources of any kind are permitted. Rely on your brain!
- Compasses, protractors, rulers, straightedges, graph paper, blank scratch paper, and writing implements are generally permitted, so long as they are not designed to give an unfair advantage.
- No computational aids (including but not limited to calculators, phones, calculator watches, and computer programs) are permitted on any portion of the MOAA.
- No individual may receive help from any other person, including members of their team. Consulting any other individual is grounds for disqualification.

How to Compete

- After completing the test, you should input your answers, along with your Team ID and name, into the provided Accuracy Round Google Form.

Special Thanks to Our Sponsors!

Accuracy Round Problems

The Accuracy Round consists of 10 problems, ordered in approximately increasing difficulty, to be solved in 45 minutes. All answers are nonnegative integers no larger than 1,000,000.

A1. Evaluate

$$2 \times (2 \times (2 \times (2 \times (2 \times (2 \times 2 - 2) - 2) - 2) - 2) - 2) - 2.$$

A2. On Andover's campus, Graves Hall is 60 meters west of George Washington Hall, and George Washington Hall is 80 meters north of Paresky Commons. Jessica wants to walk from Graves Hall to Paresky Commons. If she first walks straight from Graves Hall to George Washington Hall and then walks straight from George Washington Hall to Paresky Commons, it takes her 8 minutes and 45 seconds while walking at a constant speed. If she walks with the same speed directly from Graves Hall to Paresky Commons, how much time does she save, in seconds?

A3. Arnav is placing three rectangles into a 3×3 grid of unit squares. He has a 1×3 rectangle, a 1×2 rectangle, and a 1×1 rectangle. He must place the rectangles onto the grid such that the edges of the rectangles align with the gridlines of the grid. If he is allowed to rotate the rectangles, how many ways can he place the three rectangles into the grid, without overlap?

A4. Compute the number of two-digit numbers $\overline{ab}$ with nonzero digits a and b such that a and b are both factors of $\overline{ab}$.

A5. If x, y, z are nonnegative integers satisfying the equation below, then compute $x + y + z$.

$$\left(\frac{16}{3}\right)^x \times \left(\frac{27}{25}\right)^y \times \left(\frac{5}{4}\right)^z = 256.$$

A6. Let $\triangle ABC$ be a triangle in a plane such that $AB = 13$, $BC = 14$, and $CA = 15$. Let D be a point in three-dimensional space such that $\angle BDC = \angle CDA = \angle ADB = 90^\circ$. Let d be the distance from D to the plane containing $\triangle ABC$. The value d^2 can be expressed as $\frac{m}{n}$ for relatively prime positive integers m and n. Compute $m + n$.

A7. Jeffrey rolls fair three six-sided dice and records their results. The probability that the mean of these three numbers is greater than the median of these three numbers can be expressed as $\frac{m}{n}$ for relatively prime positive integers m and n. Compute $m + n$.

A8. Will has a magic coin that can remember previous flips. If the coin has already turned up heads m times and tails n times, the probability that the next flip turns up heads is exactly $\frac{m+1}{m+n+2}$. Suppose that the coin starts at 0 flips. The probability that after 10 coin flips, heads and tails have both turned up exactly 5 times can be expressed as $\frac{m}{n}$ for relatively prime positive integers m and n. Compute $m + n$.

A9. Let S be the set of ordered pairs (x, y) of positive integers for which $x + y \leq 20$. Evaluate

$$\sum_{(x,y) \in S} (-1)^{x+y} xy.$$

A10. In $\triangle ABC$, let X and Y be points on segment BC such that $AX = XB = 20$ and $AY = YC = 21$. Let J be the A-excenter of triangle $\triangle AXY$. Given that J lies on the circumcircle of $\triangle ABC$, the length of BC can be expressed as $\frac{m}{n}$ for relatively prime positive integers m and n. Compute $m + n$.

Note: In triangle $\triangle AXY$, the A-excenter is the intersection of the interior angle bisector of $\angle XAY$ with the exterior angle bisectors of $\angle XYA$ and $\angle YXA$.