MOAA 2021: Speed Round

October 16th, 2021

Rules

- No mathematical texts, notes, or online resources of any kind are permitted. Rely on your brain!
- Compasses, protractors, rulers, straightedges, graph paper, blank scratch paper, and writing implements are generally permitted, so long as they are not designed to give an unfair advantage.
- No computational aids (including but not limited to calculators, phones, calculator watches, and computer programs) are permitted on any portion of the MOAA.
- No individual may receive help from any other person, including members of their team. Consulting any other individual is grounds for disqualification.

How to Compete

- After completing the test, you should input your answers, along with your Team ID and name, into the provided Speed Round Google Form.

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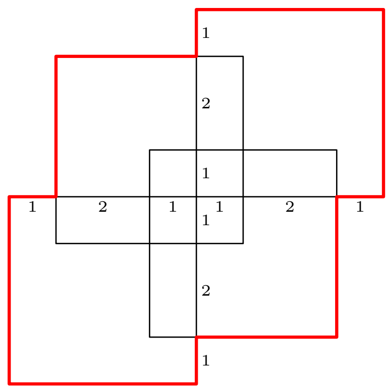
Speed Round Problems

The Speed Round consists of 10 problems, ordered in approximately increasing difficulty, to be solved in 20 minutes. All answers are nonnegative integers no larger than 1,000,000.

S1. What is $2021 + 20 + 21 + 2 + 0 + 2 + 1$?

S2. Four squares, each with area 16, are overlapped to create the following figure, with side lengths marked below.



Compute the area of the resulting shape, drawn in red above.

S3. Find the number of ordered pairs (x, y), where x and y are both integers between 1 and 9, inclusive, such that the product $x \times y$ ends in the digit 5.

S4. Let a, b, and c be real numbers such that $0 \le a, b, c \le 5$ and $2a + b + c = 10$. Over all possible values of a, b, and c, determine the maximum possible value of $a + 2b + 3c$.

S5. There are 12 students in Mr. DoBa's math class. On the final exam, the average score of the top 3 students was 8 more than the average score of the other students, and the average score of the entire class was 85. Compute the average score of the top 3 students.

S6. Suppose (a, b) is an ordered pair of integers such that the three numbers a, b, and ab form an arithmetic progression, in that order. Find the sum of all possible values of a.

S7. If positive real numbers x, y, z satisfy the following system of equations, compute $x+y+z$.

$$\begin{cases} xy + yz = 30, \\ yz + zx = 36, \\ zx + xy = 42. \end{cases}$$

S8. Andrew chooses three (not necessarily distinct) integers a, b, and c independently and uniformly at random from $\{1, 2, 3, 4, 5, 6, 7\}$. Let p be the probability that $abc(a + b + c)$ is divisible by 4. If p can be written as $\frac{m}{n}$ for relatively prime positive integers m and n, then compute $m + n$.

S9. Triangle $\triangle ABC$ has $\angle A = 90°$ with $BC = 12$. Square $BCDE$ is drawn such that A is in its interior. The line through A tangent to the circumcircle of $\triangle ABC$ intersects CD and BE at P and Q, respectively. If $PA = 4 \cdot QA$, and the area of $\triangle ABC$ can be expressed as $\frac{m}{n}$ for relatively prime positive integers m and n, then compute $m + n$.

S10. Let $ABCD$ be a unit square in the plane. Points X and Y are chosen independently and uniformly at random on the perimeter of $ABCD$. If the expected value of the area of triangle $\triangle AXY$ can be expressed as $\frac{m}{n}$ for relatively prime positive integers m and n, compute $m + n$.