MOAA 2022: Gunga Bowl

October 8th, 2022

Gunga Bowl Problems

Gunga Bowl Set 1

G1. [**8**] The Daily Challenge office has a machine that outputs the number 2.75 when operated. If it is operated 12 times, then what is the sum of all 12 of the machine outputs?

G2. [**8**] A car traveling at a constant velocity v takes 30 minutes to travel a distance of d. How long does it take, in minutes, for it travel $10d$ with a constant velocity of $2.5v$?

G3. [**8**] Andy originally has 3 times as many jelly beans as Andrew. After Andrew steals 15 of Andy's jelly beans, Andy now only has 2 times as many jelly beans as Andrew. Find the number of jelly beans Andy originally had.

Gunga Bowl Set 2

G4. [**10**] A coin is weighted so that it is 3 times more likely to come up as heads than tails. How many times more likely is it for the coin to come up heads twice consecutively than tails twice consecutively?

G5. [**10**] There are n students in an Areteem class. When 1 student is absent, the students can be evenly divided into groups of 5. When 8 students are absent, the students can evenly be divided into groups of 7. Find the minimum possible value of n.

G6. [**10**]Trapezoid $ABCD$ has $AB \parallel CD$ such that $AB = 5$, $BC = 4$ and $DA = 2$. If there exists a point M on CD such that $AM = AD$ and $BM = BC$, find CD.

Gunga Bowl Set 3

G7. [**12**] Angeline has 10 coins (either pennies, nickels, or dimes) in her pocket. She has twice as many nickels as pennies. If she has 62 cents in total, then how many dimes does she have?

G8. [**12**] Equilateral triangle ABC has side length 6. There exists point D on side BC such that the area of ABD is twice the area of ACD. There also exists point E on segment AD such that the area of ABE is twice the area of BDE. If $\mathcal{A}$ is the area of triangle ACE, then find $\mathcal{A}^2$.

G9. [**12**] A number n can be represented in base 6 as $\underline{aba}_6$ and base 15 as $\underline{ba}_{15}$, where a and b are not necessarily distinct digits. Find n.

Gunga Bowl Set 4

G10. [**14**] Let $ABCD$ be a square with side length 1. It is folded along a line ℓ that divides the square into two pieces with equal area. The minimum possible area of the resulting shape is $\mathcal{A}$. Find the integer closest to $100\mathcal{A}$.

G11. [**14**] The 10-digit number $\underline{1A2B3C5D6E}$ is a multiple of 99. Find $A+B+C+D+E$.

G12. [**14**] Let A, B, C, D be four points satisfying $AB = 10$ and $AC = BC = AD = BD = CD = 6$. If $\mathcal{V}$ is the volume of tetrahedron $ABCD$, then find $\mathcal{V}^2$.

Gunga Bowl Set 5

G13. [**16**] Nate the giant is running a 5000 meter long race. His first step is 4 meters, his next step is 6 meters, and in general, each step is 2 meters longer than the previous one. Given that his nth step will get him across the finish line, find n.

G14. [**16**] In square $ABCD$ with side length 2, there exists a point E such that $DA = DE$. Let line BE intersect side AD at F such that $BE = EF$. The area of ABE can be expressed in the form $a - \sqrt{b}$ where a is a positive integer and b is a square-free integer. Find $a + b$.

G15. [**16**] Patrick the Beetle is located at 1 on the number line. He then makes an infinite sequence of moves where each move is either moving 1, 2, or 3 units to the right. The probability that he does reach 6 at some point in his sequence of moves is $\frac{m}{n}$ where m and n are relatively prime positive integers. Find $m + n$.

Gunga Bowl Set 6

G16. [**18**] Find the smallest positive integer c greater than 1 for which there do not exist integers $0 \leq x, y \leq 9$ that satisfy $2x + 3y = c$.

G17. [**18**] Jaeyong is on the point $(0, 0)$ on the coordinate plane. If Jaeyong is on point (x, y), he can either walk to $(x + 2, y)$, $(x + 1, y + 1)$, or $(x, y + 2)$. Call a walk to $(x + 1, y + 1)$ an *Brilliant* walk. If Jaeyong cannot have two *Brilliant* walks in a row, how many ways can he walk to the point $(10, 10)$?

G18. [**18**] *Déjà vu?*

Let $ABCD$ be a square with side length 1. It is folded along a line ℓ that divides the square into two pieces with equal area. The maximum possible area of the resulting shape is $\mathcal{B}$. Find the integer closest to $100\mathcal{B}$.

Gunga Bowl Set 7

G19. [**20**] How many ordered triples (x, y, z) with $1 \leq x, y, z \leq 50$ are there such that both $x + y + z$ and $xy + yz + zx$ are divisible by 6?

G20. [**20**] Triangle ABC has orthocenter H and circumcenter O. If D is the foot of the perpendicular from A to BC, then $AH = 8$ and $HD = 3$. If $\angle AOH = 90^\circ$, find BC^2.

G21. [**20**] Nate flips a fair coin until he gets two heads in a row, immediately followed by a tails. The probability that he flips the coin exactly 12 times is $\frac{m}{n}$ where m and n are relatively prime positive integers. Find $m + n$.

Gunga Bowl Set 8

G22. **[22]** Let f be a function defined by $f(1) = 1$ and

$$f(n) = \tfrac{1}{p} f(\tfrac{n}{p}) f(p) + 2p - 2,$$

where p is the least prime dividing n, for all integers $n \geq 2$. Find $f(2022)$.

G23. **[22]** Jessica has 15 balls numbered 1 through 15. With her left hand, she scoops up 2 of the balls. With her right hand, she scoops up 2 of the remaining balls. The probability that the sum of the balls in her left hand is equal to the sum of the balls in her right hand can be expressed as $\frac{m}{n}$ where m and n are relatively prime positive integers. Find $m + n$.

G24. **[22]** Let $ABCD$ be a cyclic quadrilateral such that its diagonal $BD = 17$ is the diameter of its circumcircle. Given $AB = 8$, $BC = CD$, and that a line ℓ through A intersects the incircle of ABD at two points P and Q, the maximum area of CPQ can be expressed as a fraction $\frac{m}{n}$ for relatively prime positive integers m and n. Find $m + n$.

Gunga Bowl Set 9

This set consists of three estimation problems, with scoring schemes described.

G25. **[30]** Estimate N, the total number of participants (in person and online) at MOAA this year.

An estimate of e gets a total of $\max\left(0, \left\lfloor 150\left(1 - \frac{|N-e|}{N}\right)\right\rfloor - 120\right)$ points.

G26. **[30]** If A is the the total number of in person participants at MOAA this year, and B is the total number of online participants at MOAA this year, estimate N, the product AB.

An estimate of e gets a total of $\max(0, 30 - \lceil \log_{10}(8|N-e|+1) \rceil)$ points.

G27. **[30]** Estimate N, the total number of letters in all the teams that signed up for MOAA this year, both in person and online.

An estimate of e gets a total of $\max\left(0, 30 - \lceil 7 \log_5(|N - E|) \rceil\right)$ points.