MOAA 2022: Speed Round

October 8th, 2022

Rules

- No mathematical texts, notes, or online resources of any kind are permitted. Rely on your brain!
- Compasses, protractors, rulers, straightedges, graph paper, blank scratch paper, and writing implements are generally permitted, so long as they are not designed to give an unfair advantage.
- No computational aids (including but not limited to calculators, phones, calculator watches, and computer programs) are permitted on any portion of the MOAA.
- No individual may receive help from any other person, including members of their team. Consulting any other individual is grounds for disqualification.

How to Compete

- **In Person:** After completing the test, write your answers down in the provided Speed Round answer sheet. The proctors will collect your answer sheets immediately after the test ends.
- **Online:** After completing the test, you should input your answers, along with your Team ID and name, into the provided Speed Round Google Form.

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Speed Round Problems

Welcome to the Speed Round! The Speed Round consists of 15 problems, ordered in approximately increasing difficulty, to be solved in 40 minutes. All answers are nonnegative integers no larger than 1,000,000.

S1. [**3**] What is the value of the sum

$$2 + 20 + 202 + 2022?$$

S2. [**3**] Find the smallest integer greater than 10000 that is divisible by 12.

S3. [**4**] Valencia chooses a positive integer factor of 6^{10} at random. The probability that it is odd can be expressed in the form $\frac{m}{n}$ where m and n are relatively prime integers. Find $m+n$.

S4. [**4**] How many three digit positive integers are multiples of 4 but not 8?

S5. [**4**] At the Jane Street store, Andy accidentally buys 5 dollars more worth of shirts than he had planned. Originally, including the tip to the cashier, he planned to spend all of the remaining 90 dollars on his giftcard. To compensate for his gluttony, Andy instead gives the cashier a smaller, 12.5% tip so that he still spends 90 dollars total. How much percent tip was Andy originally planning on giving?

S6. [**5**] Let A, B, C, D be four coplanar points satisfying the conditions $AB = 16$, $AC = BC = 10$, and $AD = BD = 17$. What is the minimum possible area of quadrilateral $ADBC$?

S7. [**5**] How many ways are there to select a set of three distinct points from the vertices of a regular hexagon so that the triangle they form has its smallest angle(s) equal to $30°$?

S8. [**5**] Jaeyong rolls five fair 6-sided die. The probability that the sum of some three rolls is exactly 8 times the sum of the other two rolls can be expressed as $\frac{m}{n}$ where m and n are relatively prime positive integers. Find $m + n$.

S9. [**6**] Find the least positive integer n for there exists some positive integer $k > 1$ for which k and $k + 2$ both divide $\underbrace{11\ldots1}_{n\text{ 1's}}$.

S10. [**7**] For some real constant k, line $y = k$ intersects the curve $y = |x^4 - 1|$ four times: points A, B, C and D, labeled from left to right. If $BC = 2AB = 2CD$, then the value of k can be expressed as $\frac{m}{n}$ where m and n are relatively prime positive integers. Find $m + n$.

S11. [**8**] Let a be a positive real number and $P(x) = x^2 - 8x + a$ and $Q(x) = x^2 - 8x + a + 1$ be quadratics with real roots such that the positive difference of the roots of $P(x)$ is exactly one more than the positive difference of the roots of $Q(x)$. The value of a can be written as a common fraction $\frac{m}{n}$ where m and n are relatively prime positive integers. Find $m+n$.

S12. [**10**] Let $ABCD$ be a trapezoid satisfying $AB \parallel CD$, $AB = 3, CD = 4$, with area 35. Given AC and BD intersect at E, and M, N, P, Q are the midpoints of segments AE, BE, CE, DE, respectively, the area of the intersection of quadrilaterals $ABPQ$ and $CDMN$ can be expressed as $\frac{m}{n}$ where m, n are relatively prime positive integers. Find $m + n$.

S13. [**11**] There are 8 distinct points $P_1, P_2, \ldots, P_8$ on a circle. How many ways are there to choose a set of three distinct chords such that every chord has to touch at least one other chord, and if any two chosen chords touch, they must touch at a shared endpoint?

S14. **[12]** For every positive integer k, let $f(k) > 1$ be defined as the smallest positive integer for which $f(k)$ and $f(k)^2$ leave the same remainder when divided by k. The minimum possible value of $\frac{1}{x}f(x)$ across all positive integers $x \leq 1000$ can be expressed as $\frac{m}{n}$ for relatively prime positive integers m, n. Find $m + n$.

S15. **[13]** In triangle ABC, let I be the incenter and O be the circumcenter. If AO bisects $\angle IAC$, $AB + AC = 21$, and $BC = 7$, then the length of segment AI can be expressed as $\frac{m}{n}$ where m and n are relatively prime positive integers. Find $m + n$.