# MOAA 2023: Accuracy Round

October 7th, 2023

## Rules

- You have 45 minutes to complete 10 problems. Each answer is a nonnegative integer no greater than 1,000,000.
- If $m$ and $n$ are relatively prime, then the greatest common divisor of $m$ and $n$ is 1.
- No mathematical texts, notes, or online resources of any kind are permitted. Rely on your brain!
- Compasses, protractors, rulers, straightedges, graph paper, blank scratch paper, and writing implements are generally permitted, so long as they are not designed to give an unfair advantage.
- No computational aids (including but not limited to calculators, phones, calculator watches, and computer programs) are permitted on any portion of the MOAA.
- No individual may receive help from any other person, including members of their team. Consulting any other individual is grounds for disqualification.

## How to Compete

- **In Person:** After completing the test, write your answers down in the provided Accuracy Round answer sheet. The proctors will collect your answer sheets immediately after the test ends.
- **Online:** After completing the test, you should input your answers, along with your Team pin and name, into the provided Accuracy Round Google Form.

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## Accuracy Round Problems

A1. [**3**] Compute

$$\left(20+\frac{1}{23}\right)\cdot\left(23+\frac{1}{20}\right)-\left(20-\frac{1}{23}\right)\cdot\left(23-\frac{1}{20}\right)$$

A2. [**3**] Let $ABCD$ be a square. Let $M$ be the midpoint of $BC$ and $N$ be the point on $AB$ such that $2AN = BN$. If the area of $\triangle DMN$ is 15, find the area of square $ABCD$.

A3. [**4**] Ms. Raina's math class has 6 students, including the troublemakers Andy and Harry. For a group project, Ms. Raina randomly divides the students into three groups containing 1, 2, and 3 people. The probability that Andy and Harry unfortunately end up in the same group can be expressed in the form $\frac{m}{n}$ where $m$ and $n$ are relatively prime positive integers. Find $m+n$.

A4. [**4**] A two-digit number $\overline{ab}$ is *self-loving* if $a$ and $b$ are relatively prime positive integers and $\overline{ab}$ is divisible by $a+b$. How many *self-loving* numbers are there?

A5. [**5**] Let $k$ be a constant such that exactly three real values of $x$ satisfy

$$x-|x^2-4x+3|=k$$

The sum of all possible values of $k$ can be expressed in the form $\frac{m}{n}$ where $m$ and $n$ are relatively prime positive integers, find $m+n$.

A6. [**6**] Let $b$ be a positive integer such that 2032 has 3 digits when expressed in base $b$. Define the function $S_k(n)$ as the sum of the digits of the base $k$ representation of $n$. Given that $S_b(2032)+S_{b^2}(2032)=14$, find $b$.

A7. [**8**] Pentagon $ANDD'Y$ has $AN \parallel DY$ and $AY \parallel D'N$ with $AN = D'Y$ and $AY = DN$. If the area of $ANDY$ is 20, the area of $AND'Y$ is 24, and the area of $ADD'$ is 26, the area of $ANDD'Y$ can be expressed in the form $\frac{m}{n}$ for relatively prime positive integers $m$ and $n$. Find $m+n$.

A8. [**8**] Harry wants to label the points of a regular octagon with numbers $1, 2, \ldots, 8$ and label the edges with $1, 2, \ldots, 8$. There are special rules he must follow:

- If an edge is numbered even, then the sum of the numbers of its endpoints must also be even.
- If an edge is numbered odd, then the sum of the numbers of its endpoints must also be odd.

Two octagon labelings are equivalent if they can be made equal up to rotation, but not up to reflection. If $N$ is the number of possible octagon labelings, find the remainder when $N$ is divided by 100.

A9. [**9**] Let $\triangle ABC$ be a triangle with $AB = 10$ and $AC = 11$. Let $I$ be the center of the inscribed circle of $\triangle ABC$. If $M$ is the midpoint of $AI$ such that $BM = BC$ and $CM = 7$, then $BC$ can be expressed in the form $\frac{\sqrt{a}-b}{c}$ where $a$, $b$, and $c$ are positive integers. Find $a+b+c$.

A10. [**10**] Let $S$ be a set of integers such that if $a$ and $b$ are in $S$ then $3a-2b$ is also in $S$. How many ways are there to construct $S$ such that $S$ contains exactly 4 elements in the interval $[0, 40]$?