MOAA 2024: Accuracy Round

October 5th, 2024

Rules

- You have 45 minutes to complete 10 problems. Each answer is a nonnegative integer no greater than 1,000,000.
- If m and n are relatively prime, then the greatest common divisor of m and n is 1.
- No mathematical texts, notes, or online resources of any kind are permitted. Rely on your brain!
- Compasses, protractors, rulers, straightedges, graph paper, blank scratch paper, and writing implements are generally permitted, so long as they are not designed to give an unfair advantage.
- No computational aids (including but not limited to calculators, phones, calculator watches, and computer programs) are permitted on any portion of the MOAA.
- No individual may receive help from any other person, including members of their team. Consulting any other individual is grounds for disqualification.

How to Compete

- **In Person:** After completing the test, write your answers down in the provided Accuracy Round answer sheet. The proctors will collect your answer sheets immediately after the test ends.
- **Online:** Log into the Classtime session to access the test. Input all answers directly into the provided form. Select for the test to be handed in once you are ready.

Special Thanks to Our Sponsors!

Accuracy Round Problems

A1. [**3**] Compute

$$\frac{2024 \times 2025 - 2021 \times 2022}{2}$$

A2. [**3**] Every morning, Anthony walks at a constant speed from his dorm to the Russian School of Math for class. If he walks twice as fast as usual, he gets there 7 minutes earlier. If he walks 1 km/h slower than usual, he gets there 3.5 minutes later. To the nearest integer, how many meters is the Russian School of Math from Anthony's dorm?

A3. [**4**] The 9-digit number $\overline{20240MOAA}$ is divisible by the five smallest primes, where M, O, A are (not necessarily distinct) digits. Find the 4-digit number $\overline{MOAA}$.

A4. [**4**] A non-decreasing geometric sequence of positive integers with first term a and integer common ratio r is called n-aligned if $a, r \leq n$ and all terms of the sequence yield the same remainder when divided by n. How many 12-aligned sequences exist?

A5. [**5**] Find the number of ordered triples of positive integers (a, b, c) that satisfy

$$abc + 6a + 3b + 2c = 3ab + bc + 2ca + 2024$$

A6. [**6**] Two circles ω_1 and ω_2 are defined such that ω_1 is smaller than ω_2. Let X and Y be the intersections of the two circles. Let P be the intersection of their common external tangents, and let W be the intersection of PX and ω_1. The common external tangent closer to X touches ω_1 and ω_2 at A and B respectively, so that $PA = 75$ and $AB = 33$. Given that the quantity WX can be expressed as $\frac{m}{n}$ where m and n are relatively prime positive integers, find $m + n$.

A7. [**8**] Eric is randomly labeling each cell of a rectangular 8×10 grid with the numbers $1, 2, \ldots, 80$ such that each number is used once. A cell is called a *pit* if its number is smaller than all adjacent cells (two cells are adjacent if they share an edge). If the expected number of *pits* can be expressed as $\frac{a}{b}$ where a and b are relatively prime positive integers, find $a + b$.

A8. [**8**] In an equilateral triangle $\triangle ABC$ with area 5, let D be a point on segment AC and E be the point on ray $\overrightarrow{BC}$ not on the segment BC such that $\angle CBD = \angle CDE$ and $BD = 3ED$. The area of $\triangle CDE$ can be expressed as $\frac{a}{b}$ where a and b are relatively prime positive integers. Find $a + b$.

A9. [**9**] Suppose positive real numbers x, y satisfy

$$x\sqrt{2024 - y^2} + y\sqrt{2024 - x^2} = 2024.$$

If the maximum value of $x + 7y$ can be expressed as $a\sqrt{b}$ where a and b are positive integers and b is square-free, find $a + b$.

A10. [**10**] While doing Math Kangaroo homework, Angeline is painting a 7×7 grid where all of the squares are initially white. For every square on the grid, there is a $\frac{1}{4}$ probability that Angeline paints the square blue, a $\frac{1}{4}$ probability that she paints the square red, a $\frac{1}{4}$ probability that she paints the square purple, and a $\frac{1}{4}$ probability she leaves it white. Let P be the probability that there are more red squares than blue squares. Find the remainder when $2^{100} \cdot P$ is divided by 1000.