MOAA 2024: Speed Round

October 5th, 2024

Rules

- You have 20 minutes to complete 10 problems. Each answer is a nonnegative integer no greater than 1,000,000.
- If m and n are relatively prime, then the greatest common divisor of m and n is 1.
- No mathematical texts, notes, or online resources of any kind are permitted. Rely on your brain!
- Compasses, protractors, rulers, straightedges, graph paper, blank scratch paper, and writing implements are generally permitted, so long as they are not designed to give an unfair advantage.
- No computational aids (including but not limited to calculators, phones, calculator watches, and computer programs) are permitted on any portion of the MOAA.
- No individual may receive help from any other person, including members of their team. Consulting any other individual is grounds for disqualification.

How to Compete

- **In Person:** After completing the test, write your answers down in the provided Speed Round answer sheet. The proctors will collect your answer sheets immediately after the test ends.
- **Online:** Log into the Classtime session to access the test. Input all answers directly into the provided form. Select for the test to be handed in once you are ready.

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Speed Round Problems

S1. **[2]** Compute $2024 + 202 \times 4 + 20 \times 24 + 2 \times 24$.

S2. **[2]** Let $ABCD$ be a rectangle with $AB = 12$ and $AD = 6$. Let E be a point on segment AB such that the area of ΔAED is 24. Find the perimeter of $BCDE$.

S3. **[2]** The integers $1, 2, 3, \ldots, 19, 20$ are written in a row on a blackboard at AlphaStar Academy. Nate draws a vertical line between two consecutive numbers n and $n+1$. Given that the sum of the numbers on the left of Nate's line and the sum of the numbers on the right of his line are equal, find n.

S4. **[3]** Harry has a fair 4-sided dice while Brandon has a fair 8-sided dice. After they both roll their dice once, the probability that Brandon's number is larger than Harry's number can be expressed as $\frac{a}{b}$, where a and b are relatively prime positive integers. Find the value of $a + b$.

S5. **[3]** Let x and y be two positive integers that satisfy the following equations:

$$x + y = 33$$
$$\gcd(x, y) + \operatorname{lcm}(x, y) = 87$$

Find xy.

S6. **[4]** Let ΔABC be a triangle with $AB = 12$. The angle bisector of $\angle ABC$ intersects side AC at point D such that $AD = 8$ and $BD = CD$. Find the length of BC.

S7. **[5]** A set S contains 13 distinct positive integers, and satisfies the following two conditions:

a) The number of distinct sums $a + b$ where a, b are two (not necessarily distinct) elements of S is 25.

b) The median of S is 38.

Find the smallest possible value for an element of S.

S8. **[6]** Let x, y, z be positive real numbers satisfying

$$\frac{16 - x^2 + 2yz}{(y+z)^2} + \frac{16 - y^2 + 2xz}{(x+z)^2} + \frac{16 - z^2 + 2xy}{(x+y)^2} = 3$$

The minimum possible value of xyz can be expressed as $\frac{a\sqrt{b}}{c}$, where a, b, c are positive integers such that a and c are relatively prime and b is square-free. Find $a + b + c$.

S9. **[6]** For a set S, an element $a \in S$ is called an *atom* if there **does not** exist distinct elements $b, c \in S$ such that $a = b + c$. Let $U = \{1, 2, \cdots, 2024\}$. A subset A of U is chosen uniformly at random from all of the subsets of U (including the empty set and the full set). Suppose N is the expected number of *atoms* in A. What is the closest integer to N?

S10. **[7]** Let $P(x)$ be a polynomial with degree at least 1 that satisfies

$$P(x^2 + 1) - P(x^2 - 1) = kxP(x) + 18x^2$$

for all real numbers x where k is a constant. Find the value of $P(k)$.