

MOAA 2026 Accuracy Round Solutions

MATH OPEN AT ANDOVER

October 10, 2026

- A1. Find the last digit of

$$2^{2026} + 0^{2026} + 2^{2026} + 6^{2026}.$$

Proposed by: Paige Zhu

Answer: $\boxed{4}$

Solution: The last digit of 2 when raised to the power of 2 alternates in the pattern 2, 4, 8, 6. Because $2026 \equiv 2 \pmod{4}$, the last digit of $2^{2026} \equiv 4 \pmod{10}$. Similarly, we note that $6^{2026} \equiv 6 \pmod{10}$ and $0^{2026} = 0$. Thus, the last digit of our expression is simply $4 + 0 + 6 + 4 \equiv \boxed{4} \pmod{10}$.

- A2. Find the number of ordered pairs of positive integers (m, n) that satisfy the equation

$$\frac{1}{m} + \frac{1}{n} = \frac{1}{2026}.$$

Proposed by: Carolyn Cao

Answer: $\boxed{9}$

Solution: Multiply the entire equation by $2026mn$:

$$2026n + 2026m = mn \implies mn - 2026m - 2026n = 0$$

Add 2026^2 to both sides, then factor using Simon's Favorite Factoring Trick:

$$(m - 2026)(n - 2026) = 2026^2$$

Since m and n must be positive integers, $m - 2026$ and $n - 2026$ must be integer factors of 2026^2 . They cannot both be negative since their sum needs to yield positive m and n . The number of pairs matches the number of positive divisors of 2026^2 . Finding the prime factorization:

$$2026^2 = (2 \cdot 1013)^2 = 2^2 \cdot 1013^2.$$

The number of divisors of 2026^2 therefore is $(2 + 1)(2 + 1) = 9$, so there are $\boxed{9}$ valid ordered pairs.

- A3. Call a convex polygon *almost-rigid* if all but exactly one of its interior angles form an integer non-constant arithmetic progression. Find the maximum number of sides an almost-rigid polygon can have.

(Clarification in test: the angles do NOT have to form an arithmetic progression consecutively.)

Proposed by: Oliver Zhang

Answer: 27

Solution: We will use the fact that the exterior angles of a polygon sum to 360° . Suppose that the angles that form an arithmetic progression have corresponding exterior angles $a_1 < a_2 < \dots < a_n$. Then we need $a_1 + a_2 + \dots + a_n \leq 360$. Since $a_k \geq 1$ for all k , to maximize n we need to minimize a_1 , so let $a_1 = 1$, implying $a_k = k$. Therefore, we need

$$1 + 2 + \dots + n \leq 360 \iff \frac{n(n+1)}{2} \leq 360 \iff n \leq 26.$$

Hence, the answer is $26 + 1 = \boxed{27}$.

- A4. Bill is forming a math team from a group of 12 students. He wants to select a team of at least two students and then designate exactly two students to be the president and vice-president. Determine the number of different ways can Bill form the team.

(Clarification in test: The vice president and the president ARE on the team, and they are DIFFERENT people.)

Proposed by: Paige Zhu

Answer: 135168

Solution: Instead of selecting the team and then the two presidents, we can select the two presidents and then the rest of the team.

There are $12 \cdot 11$ ways of choosing a president and a vice-president, and there are 2^{10} ways of choosing a subset of the remaining 10 students to be on the team. Hence

$$12 \cdot 11 \cdot 2^{10} = \boxed{135168}.$$

- A5. Three vertices of a regular nonagon are chosen independently and uniformly at random, repetition allowed. If the probability that the chosen points form an acute triangle can be written as $\frac{m}{n}$ for relatively prime positive integers m and n , find $m + n$.

Proposed by: Ishan Gandhi

Answer: 101

Solution: Since the three vertices are independently chosen, there are $9^3 = 729$ total outcomes.

Notice that three distinct vertices cut the circumcircle into three arcs of a, b, c steps with $a + b + c = 9$ and $a, b, c \geq 1$. Each step is a central angle of 40° . Therefore, the inscribed angle theorem gives that the angle opposite the arc of a steps equals $20a$ degrees. Hence the triangle is acute exactly when

$$20a, 20b, 20c < 90 \implies a, b, c \leq 4$$

The only possible sets (a, b, c) are $(4, 4, 1)$, $(4, 3, 2)$, and $(3, 3, 3)$. For the first set, there are $\frac{9 \cdot 3}{3} = 9$ triangle since we may start at any of the 9 vertices, but each triangle is overcounted by a factor of 3. Similarly, the second case gives $3 \cdot 6$ and $3 \cdot 1$. Summing gives $9 + 18 + 3 = 30$ acute triangles, or $30 \cdot 3! = 180$ ordered triples. $P = \frac{180}{729} = \frac{20}{81}$, so our answer is 101.

- A6. Let $N = 2026 \cdot 2024 \cdot \dots \cdot 2$. Sophia writes N in base 11. Kaylee also writes N in a positive integer base, b . Suppose that Kaylee's base- b representation of N has at least as many trailing zeroes as Sophia's. If S is the sum of all possible values of b , find the sum of all distinct prime divisors of S .

Note: $b = 1$ is allowed, in which case Kaylee just writes a very long string of 0s.

(Clarification in test: Kaylee's base- b representation of N CAN have the same number of trailing zeroes as Sophia's.)

Proposed by: Paige Zhu and Oliver Zhang

Answer: 520

Solution: The following claim makes all computations a lot easier:

Claim: If n , m , and N are positive integers, then

$$\left\lfloor \frac{N}{nm} \right\rfloor = \left\lfloor \frac{\left\lfloor \frac{N}{n} \right\rfloor}{m} \right\rfloor.$$

Proof. Let $N = b_k n^k + b_{k-1} n^{k-1} + \dots + b_0 = \overline{b_k b_{k-1} \dots b_0}$ be the base- n expansion of N . Then

$$\frac{N}{n} = b_k n^{k-1} + b_{k-1} n^{k-2} + \dots + b_1 + \frac{b_0}{n}.$$

Let $S = b_k n^{k-1} + b_{k-1} n^{k-2} + \dots + b_1$ be the integer part of that. We then have

$$\left\lfloor \frac{N}{nm} \right\rfloor = \left\lfloor \frac{S + b_0/n}{m} \right\rfloor \quad \text{and} \quad \left\lfloor \frac{\left\lfloor \frac{N}{n} \right\rfloor}{m} \right\rfloor = \left\lfloor \frac{S}{m} \right\rfloor.$$

But $\frac{b_0}{n} < 1$, and since S and m are integers, there is no integer multiple of m that is $> S$ and $\leq S + \frac{b_0}{n}$, so $\left\lfloor \frac{S + b_0/n}{m} \right\rfloor = \left\lfloor \frac{S}{m} \right\rfloor$ and we are done. $\square$

Write $N = 2^{1013} \cdot 1013!$. Given a prime p , let $v_p(N)$ denote the largest integer power of p dividing N . It is well-known that

$$v_p(N!) = \sum_k \left\lfloor \frac{N}{p^k} \right\rfloor,$$

so by with the claim above, in reality we can compute

$$v_p(N!) = \left\lfloor \frac{N}{p} \right\rfloor + \left\lfloor \frac{\left\lfloor \frac{N}{p} \right\rfloor}{p} \right\rfloor + \left\lfloor \frac{\left\lfloor \frac{\left\lfloor \frac{N}{p} \right\rfloor}{p} \right\rfloor}{p} \right\rfloor + \dots$$

For example,

$$v_{11}(1013!) = \left\lfloor \frac{1013}{11} \right\rfloor + \left\lfloor \frac{\left\lfloor \frac{1013}{11} \right\rfloor}{11} \right\rfloor = 92 + \left\lfloor \frac{92}{11} \right\rfloor = 92 + 8 = 100.$$

Doing the same thing for 7, 5, 3, and 2 yield 166, 251, 503, and 1005, respectively. Then $v_2(N) = 1013 + v_2(1013!) = 2018$. If we write N in any base $b = p_1^{a_1} \cdot p_2^{a_2} \cdot \dots \cdot p_k^{a_k}$,

the number of trailing zeros will be the largest power of b dividing N , which is equal to the minimum across the largest power of each $p_i^{a_i}$ dividing N , or

$$\min \left\{ \left\lfloor \frac{v_{p_1}(N)}{a_1} \right\rfloor, \left\lfloor \frac{v_{p_2}(N)}{a_2} \right\rfloor, \dots, \left\lfloor \frac{v_{p_k}(N)}{a_k} \right\rfloor \right\}.$$

Since $v_p(1013!) > v_{11}(1013!)$ for any $p > 11$, we have that $b = 2^a 3^b 5^c 7^d 11^e$. By the computations above, we see that we need $0 \leq a \leq 20$, $0 \leq b \leq 5$, $0 \leq c \leq 2$, and $0 \leq d, e \leq 1$ for the above minimum to be at least 100. Then the sum of all possible values of b is

$$\begin{aligned} & (1 + 2 + \dots + 2^{20}) (1 + 3 + \dots + 3^5) \cdot (1 + 5 + 5^2) \cdot (1 + 7) \cdot (1 + 11) \\ &= (2^{21} - 1) \cdot 364 \cdot 31 \cdot 8 \cdot 12 \\ &= ((2^7)^3 - 1) \cdot (2^2 \cdot 7 \cdot 13) \cdot 31 \cdot 2^3 \cdot (2^2 \cdot 3) \\ &= (2^7 - 1) (2^{14} + 2^7 + 1) \cdot 2^6 \cdot 3 \cdot 7 \cdot 13 \cdot 31 \\ &= 127 \cdot 16513 \cdot 2^6 \cdot 3 \cdot 7 \cdot 13 \cdot 31 \\ &= 2^6 \cdot 3 \cdot 7^3 \cdot 13 \cdot 31 \cdot 127 \cdot 337 \end{aligned}$$

with prime divisors summing to $\boxed{520}$.

- A7. A compass starts with legs at $A(0,0)$ and $B(1,0)$. Each second, one leg is randomly fixed (starting with A) while the other rotates to a random adjacent lattice point. If the probability that A reaches the line $x + y = -2$ before $x + y = 2026$ can be written as $\frac{m}{n}$ for relatively prime positive integers m and n , find $m + n$.

Proposed by: Oliver Zhang

Answer: $\boxed{2027}$

Solution: As the problem suggests, let the *weight* of the compass be the sum of the coordinates of leg A . Therefore, initially the compass has weight 0.

Firstly, observe that the randomly selected fixed leg is a red herring: we may assume that the fixed leg alternates between A and B and ignore any times when the fixed leg stays the same between consecutive rounds, as the outcome of the final rotation on that leg is all that matters. Let a *round* consist of one rotation at A and one rotation at B . Notice that the weight of a compass can only change at the end of a round, never in the middle (as A is fixed in the middle).

The claim is that the problem is equivalent to a random walk on even numbers on the number line from -2 to 2026 . To see this, note that after a rotation centered at A , leg B can be either to the top/right of A or to the bottom/left of A , with equal probability. If B is to the top/right of A , the next rotation at B increases the weight by 2 or 0 with equal probability, and if B is to the bottom/left of A , the next rotation at B increases the weight by -2 or 0 with equal probability. Therefore, overall, after a round, the weight of the compass increases by 2 or decreases by 2 with probability $\frac{1}{4}$, and stays the same with probability $\frac{1}{2}$.

Let P_k be the probability that the weight of the compass reaches -2 before 2026 , given that the weight is currently k . From the relationship above, we see that

$$P_k = \frac{1}{4}P_{k-2} + \frac{1}{2}P_k + \frac{1}{4}P_{k+2} \implies P_k = \frac{1}{2}(P_{k-2} + P_{k+2})$$

This is the same equation as a random walk on even numbers. To finish, notice that the equation above implies

$$P_k - P_{k-2} = P_{k+2} - P_k,$$

so $\{P_{-2}, P_0, P_2, \dots, P_{2026}\}$ is an arithmetic sequence. Combined with $P_{-2} = 1$ and $P_{2026} = 0$, we can easily find $P_0 = \frac{1013}{1014} \implies \boxed{2027}$.

- A8. A rectangular box has integer side lengths $a \leq b \leq c$ such that its volume is equal to its surface area. Compute the sum of $a + b + c$ over all such boxes.

Proposed by: Carolyn Cao

Answer: $\boxed{281}$

Solution: Dividing by abc gives $1 = 2\left(\frac{1}{a} + \frac{1}{b} + \frac{1}{c}\right)$. Since $a \leq b \leq c$, we have $\frac{3}{a} \geq \frac{1}{a} + \frac{1}{b} + \frac{1}{c} = \frac{1}{2}$, so $a \leq 6$; also clearly $a \geq 3$ (checking $a = 1, 2$ yields no positive solutions). For each fixed a , the equation $abc = 2(ab + bc + ca)$ rearranges into a factorable form.

$$a = 3: bc = 6b + 6c \Rightarrow (b-6)(c-6) = 36, \text{ giving } (b, c) \in \{(7, 42), (8, 24), (9, 18), (10, 15), (12, 12)\}.$$

$$a = 4: bc = 4b + 4c \Rightarrow (b-4)(c-4) = 16, \text{ giving } (b, c) \in \{(5, 20), (6, 12), (8, 8)\}.$$

$$a = 5: 3bc = 10b + 10c \Rightarrow (3b-10)(3c-10) = 100, \text{ giving } (b, c) = (5, 10).$$

$$a = 6: bc = 3b + 3c \Rightarrow (b-3)(c-3) = 9, \text{ giving } (b, c) = (6, 6).$$

This yields the 10 boxes

$$(3, 7, 42), (3, 8, 24), (3, 9, 18), (3, 10, 15), (3, 12, 12),$$

$$(4, 5, 20), (4, 6, 12), (4, 8, 8), (5, 5, 10), (6, 6, 6),$$

Adding the sums of each box, we get a final sum of $\boxed{281}$.

- A9. Let ABC be a triangle with circumradius 25 and circumcircle Ω . A point P lies on Ω . The line through P perpendicular to BC meets Ω at $D \neq P$, the line through P perpendicular to CA meets Ω at $E \neq P$, and the line through P perpendicular to AB meets Ω at $F \neq P$. Suppose that $AEBDCF$ is a convex hexagon with $AD = 48$, $BE = 30$, and $CF = 40$. Find the area of $AEBDCF$.

Proposed by: Oliver Zhang

Answer: $\boxed{1475}$

Solution:

Claim: $AD \parallel BE \parallel CF$.

Proof. WLOG, it suffices to show that $AD \parallel BE$. We have

$$\angle ABE = \angle APE = 90^\circ - \angle PAC$$

and

$$\angle DAB = \angle DCB = 90^\circ - \angle PDC.$$

But $\angle PAC = \angle PDC$, so $\angle ABE = \angle DAB$, implying the result. $\square$

Now, let O be the center of Ω , and let the line through O perpendicular to AD , BE , and CF by the claim above intersect $\overline{AD}$, $\overline{BE}$, and $\overline{CF}$ at X , Y , and Z , respectively. By the Pythagorean Theorem, we get $OX = 7$, $OY = 20$, and $OZ = 15$, so $XY = 13$ and $XZ = 22$. Then

$$\begin{aligned} [AEBDCF] &= [AEBD] + [ADCF] \\ &= \frac{1}{2} \cdot (30 + 48) \cdot 13 + \frac{1}{2} \cdot (48 + 40) \cdot 22 \\ &= 39 \cdot 13 + 88 \cdot 11 = \boxed{1475}. \end{aligned}$$

- A10. A hunter and rabbit play a game on an infinite square lattice. The hunter starts at the origin and the rabbit starts at a point (p, q) , where $1 \leq p, q \leq 2026$. In the n -th round of the game, two things occur in order:

- The rabbit moves either up, left, down, or right exactly 1 unit.
- The hunter moves directly towards the rabbit along a straight line until it hits a grid point distinct from its starting point.

The rabbit is considered “caught” when the hunter moves to the same point as the rabbit. Let N be the number of starting points the rabbit can choose for which there exists a strategy to ensure it will never be caught. Find the remainder when N is divided by 1000.

Proposed by: Oliver Zhang

Answer: $\boxed{363}$

Solution: Call a point *safe* if the rabbit can escape from the hunter, and *deadly* otherwise.

Claim 1: All points $(k, 1)$ and $(1, k)$, where $k \neq 1$, are safe.

Proof. WLOG let the rabbit be at $(k, 1)$. Move to $(k, 0)$ in the first round. Observe that the hunter and rabbit are now on the same grid line (namely, the x -axis), and the hunter is at least 2 units behind the rabbit. The rabbit can simply keep moving to the right forever, as the hunter can only move 1 unit to the right each round. $\square$

Claim 2: All points $(2, 2k)$ and $(2k, 2)$, where k is a positive integer, are deadly.

Proof. Firstly observe that $(2, 2)$ is deadly, because the rabbit can only move to $(1, 2)$, $(3, 2)$, $(2, 1)$, or $(2, 3)$, and then the hunter immediately captures it. It now suffices to prove that all points $(2, 2k)$ are deadly.

If $k > 1$, suppose that all points $(2, 2r)$ where $1 \leq r < k$ are deadly. Starting at $(2, 2k)$, the rabbit can move to $(1, 2k)$, $(2, 2k - 1)$, $(2, 2k + 1)$, where the hunter catches it immediately, or $(3, 2k)$. If $k \not\equiv 0 \pmod 3$ the hunter also catches it immediately, and if $k \equiv 0 \pmod 3$ then the hunter moves to $(1, \frac{2k}{3})$. Now, the rabbit is at $(2, \frac{4k}{3})$ relative to the hunter. Since $\frac{4k}{3} < 2k$, we are done by induction. $\square$

Claim 3: All points (x, y) with $x, y > 1$ and $x \equiv y \pmod 2$ are deadly.

Proof. In the first move, the rabbit moves to some point (w, z) , where $w \not\equiv z \pmod 2$. WLOG let w be even and z be odd. If $\gcd(w, z) = 1$ then the hunter immediately captures the rabbit, so suppose $\gcd(w, z) > 1$. The hunter moves to $(\frac{w}{\gcd(w, z)}, \frac{z}{\gcd(w, z)})$, so the rabbit will be at $(w - \frac{w}{\gcd(w, z)}, z - \frac{z}{\gcd(w, z)})$ relative to the hunter after the round. However, note that $\gcd(w, z)$ is odd, so $\frac{w}{\gcd(w, z)}$ is even

and $\frac{z}{\gcd(w,z)}$ is odd. Therefore, relative to the hunter, both coordinates of the rabbit are even.

Now, by repeating this process, since both coordinates of the rabbit relative to the hunter strictly decrease after each round and also both stay even, eventually the rabbit must reach some point $(2, 2k)$ or $(2k, 2)$ relative to the hunter, where it will eventually get captured by the previous claim. $\square$

Claim 4: All points (x, y) with $x, y > 1$ and $x \not\equiv y \pmod{2}$ are safe.

Proof. WLOG let x be even and y be odd. Let $v_2(N)$ denote the power of 2 in the prime factorization of N . Since $v_2(y-1) \neq v_2(y+1)$, either $v_2(y-1) \neq v_2(x)$ or $v_2(y+1) \neq v_2(x)$. WLOG let $v_2(y-1) \neq v_2(x)$. Let the rabbit move to $(x, y-1)$, and suppose $x = 2^a r$, $y-1 = 2^b s$ with $a \neq b$ and r, s odd. WLOG suppose $a < b$. Then $\gcd(x, y-1) = 2^a \gcd(r, s)$, so the hunter moves to

$$\left(\frac{2^a r}{2^a \gcd(r, s)}, \frac{2^b s}{2^a \gcd(r, s)} \right) = \left(\frac{r}{\gcd(r, s)}, 2^{b-a} \cdot \frac{s}{\gcd(r, s)} \right).$$

Then, relative to the hunter, the rabbit will be at

$$\left(2^a r - \frac{r}{\gcd(r, s)}, 2^b s - 2^{b-a} \cdot \frac{s}{\gcd(r, s)} \right).$$

Observe that $\frac{r}{\gcd(r, s)}$ and $\frac{s}{\gcd(r, s)}$ are both odd, and $b-a > 0$ implies that $2^{b-a} \cdot \frac{s}{\gcd(r, s)}$ is even, so the rabbit is now at a point (odd, even) relative to the hunter.

Again, by repeating this process, both coordinates of the rabbit relative to the hunter strictly decrease and stay (odd, even), so eventually the rabbit must reach some point $(1, k)$ or $(k, 1)$, from where it can escape from the hunter by the first claim. $\square$

Finally, combining all the claims above, we know exactly which points are deadly. Now we use complementary counting: suppose $x \equiv y \pmod{2}$, where $1 < x, y \leq 2026$. There are 1013 even numbers and 1012 odd numbers in this range, so there are $1013^2 + 1012^2$ such pairs (x, y) . There are 2026^2 points the rabbit can choose to start on, so the total number of safe points is

$$2026^2 - 1013^2 - 1012^2 \equiv 26^2 - 13^2 - 12^2 \equiv \boxed{363} \pmod{1000}.$$