

MOAA 2026 Speed Round Solutions

MATH OPEN AT ANDOVER

October 10th, 2026

- S1. Compute $20 \cdot 26 + 202 - 6$.

Proposed by: Paige Zhu

Answer: 716

Solution: Straightforward calculation gives

$$20 \times 26 + 202 - 6 = 520 + 196 = \boxed{716}.$$

- S2. Notebooks at Areteem are sold in two pack sizes: a pack of 3 notebooks costs \$4, and a pack of 5 notebooks costs \$6. If Carolyn can only buy a whole number of packs, determine the minimum number of dollars she must spend to buy *at least* 37 notebooks.

Proposed by: Carolyn Cao

Answer: 46

Solution: Since a 5-pack costs \$1.20 per notebook versus $\$1.\bar{3}$ per notebook for a 3-pack, 5-packs are better value. If Paige buys a packs of 5 and b packs of 3, she needs $5a + 3b \geq 37$ and wants to minimize $6a + 4b$. Checking values of a from 0 to 8 and taking the smallest valid b for each:

a	$b_{\min}$	$5a + 3b$	cost = $6a + 4b$
7	1	38	46
5	4	37	46
6	3	39	48
8	0	40	48

No combination beats 46.

- S3. The product of three consecutive positive integers is equal to 33 times their sum. Find the largest of the three integers.

Proposed by: Carolyn Cao

Answer: 11

Solution: Let the integers be $n-1$, n , $n+1$. Their product is $n^3 - n = n(n-1)(n+1)$ and their sum is $3n$. The condition gives

$$n(n^2 - 1) = 33(3n) = 99n.$$

Since $n > 0$, divide both sides by n : $n^2 - 1 = 99$, so $n^2 = 100$ and $n = 10$. The three integers are 9, 10, 11, and indeed $9 \cdot 10 \cdot 11 = 990 = 33 \cdot 30$. Thus, the largest of the three integers is 11.

- S4. Let $ABCD$ be a rectangle with $AB = 15$ and $BC = 20$. The incircle of $\triangle ABC$ touches sides $\overline{AB}$ and $\overline{BC}$ at points X and Y , respectively. If the circumcircle of $\triangle DXY$ has area $K\pi$, find K .

Proposed by: Oliver Zhang

Answer: 125

Solution: Let I be the incenter of $\triangle ABC$. $AC = 25$, so $\triangle ABC$ has semiperimeter $s = \frac{1}{2} \cdot (15 + 20 + 25) = 30$. Also, $[ABC] = \frac{1}{2} \cdot 15 \cdot 20 = 150$, so the inradius of $\triangle ABC$ is $\frac{[ABC]}{s} = 5$. Since $IXBY$ is a square, we see that $XB = BY = r = 5$, and then $CY = 15$, $AX = 10$. Therefore, by Pythagorean's Theorem, $XY = 5\sqrt{2}$, $YD = 15\sqrt{2}$, and $DX = 10\sqrt{5}$.

Now, $\triangle DXY$ has area

$$[ABCD] - [BXY] - [CYD] - [DAX] = 300 - \frac{25}{2} - \frac{225}{2} - 20 = 75,$$

so since $R = \frac{abc}{4[ABC]}$ for any triangle $\triangle ABC$, we find the circumradius of $\triangle DXY$ to be

$$R = \frac{5\sqrt{2} \cdot 15\sqrt{2} \cdot 10\sqrt{5}}{4 \cdot 75} = 5\sqrt{5},$$

and $\pi \cdot R^2 = 125\pi \implies \boxed{125}$.

- S5. Eugenia and Niki are eating letter cupcakes that spell out "JANESTREET". If they both eat five cupcakes at random, the probability that Eugenia eats all of the cupcakes with the letter "E" can be written as $\frac{m}{n}$, where m and n are relatively prime positive integers. Find $m + n$.

Proposed by: Paige Zhu

Answer: 13

Solution: We only consider the ways Eugenia selects her cupcakes. Without any restrictions, there are $\binom{10}{5}$ ways for her to choose her 5 cupcakes. If she were to eat all three cupcakes labeled "E," she must choose those three cupcakes in addition to 2 of the remaining 7, so there are $\binom{7}{2}$ desired cases. Hence, the probability is

$$\frac{\binom{7}{2}}{\binom{10}{5}} = \frac{21}{252} = \frac{1}{12},$$

and $1 + 12 = \boxed{13}$.

- S6. Let

$$S = (1^3 + 3^3 + 5^3 + \cdots + 2025^3) + 2026^3.$$

Find the remainder when S is divided by 1000.

Proposed by: Ishan Gandhi

Answer: 529

Solution: Split the cubes of 1 through $2n$ into odds and evens. Note that

$$1^3 + 2^3 + \cdots + (2n)^3 = \left(\frac{2n(2n+1)}{2}\right)^2 = n^2(2n+1)^2.$$

Factoring 8 out of the even terms gives

$$2^3 + 4^3 + \cdots + (2n)^3 = 8(1^3 + 2^3 + \cdots + n^3) = 8\left(\frac{n(n+1)}{2}\right)^2 = 2n^2(n+1)^2.$$

Subtracting gives

$$1^3 + 3^3 + \cdots + (2n-1)^3 = n^2((2n+1)^2 - 2(n+1)).$$

Applying this formula on $n = 1013$ yields

$$1^3 + 3^2 + \cdots + 2025^3 = 1013^2(2 \cdot 1013^2 - 1)$$

which is $13^2(2 \cdot 169 - 1) = 169 \cdot 337 \pmod{1000} \equiv 953 \pmod{1000}$. Adding the leftover term 2026^3 finishes the problem through computation: $953 + 2026^3 \equiv 953 + 26^3 = 953 + 17576 \equiv 953 + 576 \pmod{1000} \equiv \boxed{529} \pmod{100}$

- S7. Ten chairs are placed in a circle. The five MOAA tournament directors are to select their seats, one after the other. A chair is considered available if it is empty and both its adjacent chairs are also empty. Each person will randomly select an available chair to sit in, and they will not sit down if there are no available chairs left. The probability that all five people sit down can be written as $\frac{m}{n}$, where m and n are relatively prime positive integers. Find $m + n$.

Proposed by: Oliver Zhang

Answer: $\boxed{139}$

Solution: Let the chairs be numbered $1, 2, \dots, 10$ in order around the circle. For five people to successfully sit down, they must occupy alternating chairs, which means they will either occupy all five even-numbered chairs or all five odd-numbered chairs.

The first person (P1) can sit in any of the ten chairs with probability 1. WLOG, assume that P1 sits in chair 1. For any five people to sit, the next four must choose exclusively from 3, 5, 7, and 9. After P1 sits, we can casework P2's choice as follows:

Case 1: P2 chooses an odd chair adjacent to the occupied block (chair 3 or 9).

There are 2 such chairs out of 7 available, so the probability of this is $\frac{2}{7}$. WLOG, let P2 choose 3. The available chairs are now 5, 6, 7, 8, 9. P3 must choose an odd chair:

- If P3 chooses 5 or 9 (probability $\frac{2}{5}$), there will be 3 available chairs left, 2 of which are odd. P4 chooses one of the odd chairs with probability $\frac{2}{3}$, and P5 takes the last remaining valid chair with probability 1. Thus, this case has probability $\frac{2}{7} \cdot \frac{2}{5} \cdot \frac{2}{3} \cdot 1 = \frac{8}{105}$.
- If P3 chooses 7 (probability $\frac{1}{5}$), the only available chairs left are 5 and 9. P4 and P5 will be forced to take them, each with probability 1. This case has probability $\frac{2}{7} \cdot \frac{1}{5} \cdot 1 \cdot 1 = \frac{2}{35} = \frac{6}{105}$.

Case 2: P2 chooses an odd chair not adjacent to the occupied block (chair 5 or 7).

There are 2 such chairs out of 7 available, so the probability of this is $\frac{2}{7}$. WLOG, let P2 choose 5. The available chairs are now 3, 7, 8, 9. P3 must choose an odd chair:

- If P3 chooses 3 (probability $\frac{1}{4}$), the available chairs are 7, 8, 9. P4 must choose 7 or 9 (probability $\frac{2}{3}$), and P5 takes the last chair with probability 1, for a probability of $\frac{2}{7} \cdot \frac{1}{4} \cdot \frac{2}{3} \cdot 1 = \frac{1}{21} = \frac{5}{105}$.
- If P3 chooses 7 or 9 (probability $\frac{2}{4} = \frac{1}{2}$), exactly two valid chairs remain available, which P4 and P5 will take with probability 1. It follows that this case has probability $\frac{2}{7} \cdot \frac{1}{2} \cdot 1 \cdot 1 = \frac{1}{7} = \frac{15}{105}$.

Summing the probabilities of all cases, we get the total probability that all five people sit down is

$$P = \frac{8}{105} + \frac{6}{105} + \frac{5}{105} + \frac{15}{105} = \frac{34}{105}$$

Since $\gcd(34, 105) = 1$, we have $m = 34$ and $n = 105$. Therefore, $m + n = 34 + 105 = \boxed{139}$.

- S8. Let $ABCD$ be a nondegenerate quadrilateral with integer side lengths that satisfies $AB + CD = BC + DA$, and $AB \cdot CD = BC \cdot DA = [ABCD]$. Given that the longer diagonal of $ABCD$ has length 37, find the perimeter of $ABCD$.

Proposed by: Oliver Zhang and Pranav Mallina

Answer: $\boxed{94}$

Solution: Let $AB \cdot CD = BC \cdot DA = r$ and $AB + CD = BC + DA = s$. Then $\{AB, CD\}$ and $\{BC, DA\}$ are both solution sets to the quadratic $x^2 - sx + r = 0$, so they must be the same set. This implies that $ABCD$ is a kite.

Since $ABCD$ is a kite, it must have perpendicular diagonals, so $AC \cdot BD = 2[ABCD]$. However, by Ptolemy's inequality,

$$AB \cdot CD + BC \cdot DA \geq AC \cdot BD = 2[ABCD],$$

with equality if and only if $ABCD$ is cyclic. By the problem statement we see that equality holds, so $ABCD$ is cyclic.

By the two results above, we see that $ABCD$ is a right kite. Hence, $AB^2 + BC^2 = 37^2$. The only Pythagorean triple with 37 as the hypotenuse is $(12, 35, 37)$, so the perimeter of $ABCD$ is $2 \cdot (12 + 35) = \boxed{94}$.

- S9. Find the number of ordered pairs of positive integers (m, n) such that $m + n = 2026$ and $\gcd(m, n) = \gcd(m + 2, n + 2)$.

Proposed by: Ishan Gandhi

Answer: $\boxed{1340}$

Solution: Since $m + n = 2026$, we have $\gcd(m, n) = \gcd(m, 2026)$. Also, $(m + 2) + (n + 2) = 2030$, so it follows that $\gcd(m + 2, n + 2) = \gcd(m + 2, 2030)$. Thus we look for $\gcd(m, 2026) = \gcd(m + 2, 2030)$. Now,

$$2026 = 2 \cdot 1013, \quad 2030 = 2 \cdot 5 \cdot 7 \cdot 29.$$

Since 1013 does not divide 2030, the equality can hold only if $1013 \nmid m$. Therefore the condition reduces to requiring that

$$5 \nmid (m + 2), \quad 7 \nmid (m + 2), \quad 29 \nmid (m + 2).$$

For $1 \leq m \leq 2025$, inclusion-exclusion gives $405 + 289 + 69 - 57 - 13 - 9 + 1 = 685$ bad values of m . Hence the number of good values is $2025 - 685 = 1340$. Therefore, the number of valid ordered pairs is $\boxed{1340}$.

- S10. Triangle ABC has $AB = 20$ and $BC = 26$. Points X and Y are chosen on segments $\overline{AB}$ and $\overline{BC}$, respectively, such that $AX = CY = 2$. The perpendicular bisectors of AY and CX meet at P . Given that $PB = 24$, find $PA^2 + PC^2$.

Proposed by: Oliver Zhang

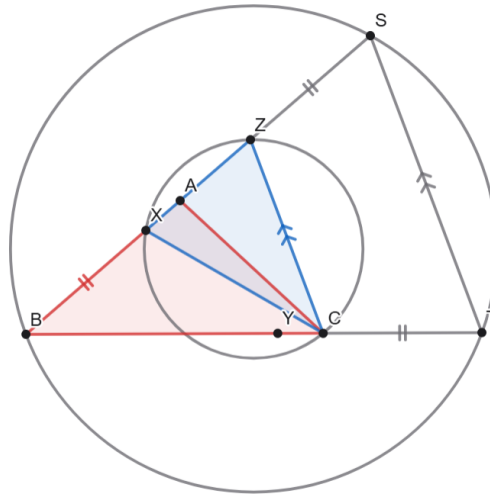
Answer: 204

Solution: Note that $AB + BY = CB + BX = 44$. Hence, we can extend BA and BC to S and T , respectively, such that $BS = BT = 44$. We claim that the perpendicular bisectors of AY and CX both pass through the circumcenter of $\triangle BST$.

WLOG, it suffices to prove this for CX . Let the line through C parallel to ST intersect segment BS at Z . Since both $\triangle BST$ and $\triangle BZC$ are isosceles, we have that CZ and ST share a perpendicular bisector, namely the angle bisector of $\angle B$. Furthermore, $CZST$ is an isosceles trapezoid, so $ZS = CT$. But

$$CT = 44 - CB = CB + BX - CB = BX,$$

so $ZS = BX$. Therefore, XZ and BS share a midpoint, so they share a perpendicular bisector as well. Hence, $\triangle CYZ$ and $\triangle BST$ share a circumcenter. Since CX passes through the circumcenter of $\triangle CYZ$, this implies the result.



To finish, let M and N be the feet from P to BS and BT , respectively. Since P is the circumcenter of $\triangle BST$, we have $BM = MS = 22$ and $BN = NT = 22$. Hence $MA = 2$ and $NC = 4$. We then obtain $PN = PM = \sqrt{PB^2 - BM^2} = \sqrt{24^2 - 22^2} = \sqrt{92}$, so $PA^2 = PM^2 + MA^2 = 96$, $PC^2 = PN^2 + NC^2 = 108$, and $96 + 108 = \boxed{204}$.